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A Study on Double Acceptance Sampling Plans based on Type II Generalized Half-Logistic Distribution

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Abstract: **This paper considers the Type II generalized half-logistic distribution (GHLD) with known shape parameter in developing double acceptance sampling based on truncating the inspection time. The proposed double sampling plan yields the best minimum inspection time and provides minimum error in terms of accepting a bad lot. We have constructed two tables using the Type II GHLD, one is obtained by considering one sample size and the other one is obtained using the two sample sizes method. An illustration was given to show the strength of double sampling over the single sampling plan in terms of decision mechanism. Finally, a study on these tables was made to make illustrations more visible using the proposed sampling plan developed here.**

Keywords: **Type II generalized half-logistic distribution, double sampling plans, truncated life test, probability of acceptance, inspection time.**

1. INTRODUCTION

Quality of a product has been the major concern to both developing and developed countries. Controlling and improving quality therefore has become an important business strategy for many organizations; manufacturers, distributors, transportation companies, financial services organizations; health care providers, and government agencies. Quality is a competitive advantage. A business that can delight customers by improving and controlling quality can dominate its competitors. Acceptance sampling as one of the important field in statistical quality control, it is concerned with inspection procedure used to determine whether to accept or reject a specific quantity of material. The methods of statistical acceptance sampling is one of the most interesting and useful applications of modern mathematical statistics (John 1947).

Rosaiah *et al.* (2014) developed an economic reliability test plan using Type II generalized half logistic distribution. In their proposed plan, it is assumed that the normal distribution is not a good fit to the data under consideration, as such inferences derive from such plan might be misleading. They determine the termination time of the experiment for a given minimum number of sample size considering single sampling plan. Works under this regards are the likes of Voda (1972), Mukherjee and Saran (1984), Mukherjee and Maiti (1997), Aslam and Jun (2009) as well as Fan and China (2015) have all developed the sampling plans based on single acceptance sampling.

This served as motivation to developing double acceptance sampling based on Type II half logistic distribution to become as extension to that of Rosaiah *et al* (2014). These kinds of sampling plans are suggested by Kantam et al (2006) and Rosaiah et al (2007) for log logistic and exponential log logistic distribution respectively. As we all know, it has proven that the double acceptance sampling plan is more efficient than the single sampling plan in terms of the sample size required. (Sudamani and Sutharani, 2013).

The problem considered here is that of finding the best operating characteristics which yields assurance that a certain average life will meet required life time when the life test is terminated at a pre-assigned time *t* and when the observed number of failures does not exceed a given acceptance number. The decision procedure is to accept a lot only if the specified average life can be established with a pre-assigned high probability *p* which provides the protection to the consumer. The decision to accept the lot can take place only at the end of time *t* and only if the number of failures does not exceed the given acceptance number *c*. The life test experiment is terminated at the time at which the $(c + 1)^{st}$ failure is observed or at time *t* whichever is earlier, in the first case the decision is to reject the lot.

2. MATERIAL AND METHODS

Consider a series system of θ components with individually and identically distributed (iid) individual lifetimes, for example, $F(x)$. The reliability function of such a system is given by $\left[1-F(x)\right]^{\theta}$; hence, the distribution function of the lifetime random variable of a series system is $1-\left[1-F(x)\right]^{\theta}$. Taking $F(x)$ as the standard half logistic model, Kantam *et al.* (2014) proposed a new model called the standard Type-II generalized half logistic distribution (GHLD), The probability density function and cumulative distribution function of Type II generalised half logistic distribution are respectively given by

$$f(x;\theta)=\frac{2^{\theta}\theta e^{x}}{\left(1+e^{x}\right)^{\theta+1}};\qquad 0\le x<\infty,\ \theta>0 \tag{1}$$

$$F(x;\theta)=1-\frac{2^{\theta}}{\left(1+e^{x}\right)^{\theta}}\ ;\qquad 0\le x<\infty,\ \theta>0 \tag{2}$$

where θ is called the shape parameter. Here, it may be noted that the above standard Type II GHLD is same as the standard Type I GHLD proposed by Olapade (2014). However, Olapade (2014) derived standard Type I GHLD with a different theoretical argument namely if Y follows an exponential distribution with parameter θ, then X= log(2e^{Y}-1) is a standard Type I GHLD with parameter θ.

3. DOUBLE ACCEPTANCE SAMPLING PLAN (DASP)

The DASP is used to minimise both the risk of the producer and consumer, because it provides another opportunity for acceptance in case the rejection occur in single sampling plan. The consumer's risk which is the probability of accepting a bad lot when in the actual sense is supposed to be accepted is minimised too under double sampling plan. Let $p*$ be a minimum confidence level with which a lot having a true average life below is rejected by the sampling plan. For a given $p*$ our sampling plan is characterized by four parameters (n_1,n_2,c_1,c_2) as will be explain later. We consider sufficiently large lots so that the binomial distribution can be applied. The interest is to determine for a given values of $p*,\sigma_0,\ c_1,$ and c_2 the smallest positive integer *n* such that

$$P(A)=C_0^{n_1}p^0q^{n_1}+C_1^{n_1}p^1q^{n_1-1}\left[\sum_{i=0}^{1}C_i^{n_2}p^iq^{n_2-i}\right]\times C_2^{n_1}p^2q^{n_1-2}\left[C_0^{n_2}p^0q^{n_2}\right]\le 1-p* \tag{3}$$

where $p=F(t;\sigma)=F(\frac{t}{\sigma_0}*\frac{\sigma_0}{\sigma})$, is given by (1) indicates the failure before time t which depends only on the ratio $\frac{t}{\sigma}$ and is sufficient to specify this ratio for designing the experiment.

We propose the following DASP procedure based on truncated life test:

1. Draw the 1st sample of size n_1 and put them on test during time t

2. Accept the lots if there are no more than c_1 failures, else reject the lot if there are more than c_2 failures.

3. If the number of failures is between c_1 and c_2, then draw the 2nd sample of size n_2 and put them on test during time t.

4. Accept the lot if the total number of failures is not more than c_2 during time t.

The DASP comprises of four parameters (n_1, n_2, c_1, c_2) if time t_0 is specified. Here n_1 and n_2 refers to 1st and 2nd sample, while c_1 and c_2 refers to the acceptance numbers associated with the 1st and 2nd sample, respectively. Let σ be the unknown average life and σ_0 be the specified average life in generalized half logistic distribution. A lot is considered good and worthy of acceptance if the true unknown average life is more than the specified average life. Likewise, the lot is considered bad if the true unknown average life is less than the specified average life.

Consider a life testing experiment having n_1 items in the 1st sample with an acceptance number $c_1 = 0$ and n_2 items in the 2nd sample with the corresponding acceptance number $c_2 = 2$. If no failure occurs in the 1st sample items put on test, we accept the lot. Probability of acceptance for these is calculated and placed in Table 1. If there is difference between σ (the true unknown lifetime average) and that of the σ_0 (the specified lifetime average) of the product, it can be obtained by probability of acceptance $L(p_1)$ and $L(p_2)$ for sampling plans $\left(n_1, c_1, t/\sigma_0\right)$ and $\left(n_2, c_2, t/\sigma_0\right)$ respectively.

For the Type II generalized half logistic distribution, the probability of acceptance (PA) is given by

$$L(p_1) = \sum_{i=0}^{c=0} \binom{n_1}{i} \left(1 - \frac{2^\theta}{\left(1+e^x\right)^\theta}\right)^i \left(\frac{2^\theta}{\left(1+e^x\right)^\theta}\right)^{n_1 - i} \qquad (4)$$

$$L(p_2) = \sum_{i=1}^{c=2} \binom{n_2}{i} \left(1 - \frac{2^\theta}{\left(1+e^x\right)^\theta}\right)^i \left(\frac{2^\theta}{\left(1+e^x\right)^\theta}\right)^{n_2 - i} \qquad (5)$$

The producer's risk is the probability of rejecting the lot when $\sigma > \sigma_0$.

The probability of acceptance for a DASP is calculated and placed in Table 1 using 4 and 5.

The minimum values of n satisfying equation 3 were obtained for $1-\alpha$ = 0.75, 0.9, 0.95, 0.99 and t/σ = .07, 0.50, 2.90, 4.99, 5.02, 6.46, 8.06, 8.93, these choices are consistent with SD. Jilani (2018) and Zoramawa (2018).

4. ILLUSTRATIVE EXAMPLE

Consider the following sorted sample of size 10, which is simulated from Type II GHLD with the scale parameter σ =10 and shape parameter θ =3 from SD. Jilani (2018).

0.07, 0.50, 2.90, 4.99, 5.02, 6.46, 8.06, 8.93, 9.46, 10.52.

The authors use the above values in obtaining the operating characteristics of double sampling plan. This is with the view to compare our proposed plan with other plans like that of Aslam (2009) and Zoramawa et al (2018) where a similar work was carried out considering the Rayleigh distribution and Inverse Rayleigh distribution respectively.

Suppose that the lifetime of a product follows the Type II generalized half logistic distribution with a shape and scaled parameter $\theta = 3$ and $\sigma = 10$ respectively and an experimenter wants to establish that its true unknown mean life is at least 1000 hours with confidence 0.95. The acceptance numbers for this experiment are $c_1 = 1$ and $c_2 = 3$ with sample sizes $n_1 = 9$ and $\mathrm{n}_2 = 11$. The lot is accepted if during 520 hours there is at most one failure is observed in a sample of 9. The probability of acceptance for this from the single point of view is from Table 2 is 0.0637. The probability of acceptance for the same setup from the double sampling point of view from Table 1 is 0.0011. In a double sampling plan as the scaled parameter ratio increases so shall the probability of acceptance increases too. For the above sampling plan, the probability of acceptance is 0.9991 when the ratio of the unknown average lifetime to the specified average lifetime is 14. But if the time of the inspection increases, the probability of acceptance for a double sampling decreases as that result. For instance, at 90% confidence the lot with sample size 13 in a single sampling plan and at 290 hours the probability of acceptance is 0.6950, but considering the same lot in a double acceptance sampling the probability of acceptance is 0.1562. One can easily notice that with the double sampling scheme it is somehow difficult to accept a bad lot in a truncated life test. Such a sample can only be accepted when the system double the ratio, as it can be seen when the ratio is doubled the probability becomes 0.9581.

The producer's risk decreases as the quality level of the product increases indicating a kind of pressure like condition which is alright since both the consumer and producer's risk is now been protected.

5. CONCLUSION

Product's lifetime is of great importance in terms of product life and quality. It is common knowledge that the consumers often prefer products that have a tendency of having a longer lifetime. This paper was able to demonstrate the power of double sampling plan over single sampling plan when the lifetime of a product follows type II GHLD.

Table 1: Operating characteristics values for the double sampling plan when $c_1 = 1$ and $c_2 = 3$

P	n1	n2	t/s	2	4	6
0.75	11	22	0.07	1.0000	1.0000	1.0000
	9	18	0.5	1.0000	1.0000	1.0000
	8	16	2.9	0.3214	0.9788	1.0000
	12	24	4.9	0.0002	0.0492	0.5154
	7	14	5.02	0.0051	0.1605	0.6967
	6	12	6.46	0.0010	0.0379	0.2677
	5	10	8.06	0.0005	0.0140	0.1025
	5	10	8.93	0.0002	0.0063	0.0526
0.90	n1	n2	t/s	2	4	6
	21	42	0.07	1.0000	1.0000	1.0000
	17	34	0.5	1.0000	1.0000	1.0000
	12	24	2.9	0.1562	0.9559	0.9999
	9	18	4.9	0.0015	0.1090	0.6474
	8	16	5.02	0.0024	0.1191	0.6421
	5	10	6.46	0.0033	0.0669	0.3523
	3	6	8.06	0.0105	0.0814	0.2885
	2	4	8.93	0.0341	0.1471	0.3690
0.95	n1	n2	t/s	2	4	6
	25	30	0.07	1.0000	1.0000	1.0000
	18	22	0.5	1.0000	1.0000	1.0000
	14	19	2.9	0.1295	0.9581	0.9999
	10	12	4.9	0.0007	0.1053	0.6908
	9	11	5.02	0.0011	0.1117	0.6773
	5	10	6.46	0.0033	0.0669	0.3523
	4	6	8.06	0.0023	0.0359	0.1963
	3	5	8.93	0.0061	0.0523	0.2060
0.99	n1	n2	t/s	2	4	6
	30	35	0.07	1.0000	1.0000	1.0000
	22	27	0.5	1.0000	1.0000	1.0000
	13	18	2.9	0.1536	0.9627	0.9999
	10	15	4.9	0.0007	0.0918	0.6510
	9	13	5.02	0.0011	0.1004	0.6470
	5	10	6.46	0.0033	0.0669	0.3523
	4	6	8.06	0.0023	0.0359	0.1963
	3	5	8.93	0.0061	0.0523	0.2060

Table 2: Operating characteristics values for the single sampling plan when $c = 0$

P	n	t/s	2	4	6	8
0.75	9	0.07	1.0000	1.0000	1.0000	1.0000
	8	0.5	1.0000	1.0000	1.0000	1.0000
	7	2.9	0.9272	0.9999	1.0000	1.0000
	6	4.9	0.3187	0.8705	0.9943	0.9999
	5	5.02	0.4439	0.9071	0.9957	0.9999
	4	6.46	0.3810	0.7945	0.9663	0.9972
	4	8.06	0.2095	0.5682	0.8501	0.9669
	3	8.93	0.4522	0.7419	0.9100	0.9768
0.90	n	t/s	2	4	6	8
	17	0.07	1.0000	1.0000	1.0000	1.0000
	15	0.5	1.0000	1.0000	1.0000	1.0000
	13	2.9	0.6950	0.9993	1.0000	1.0000
	10	4.9	0.0458	0.6069	0.9721	0.9996
	9	5.02	0.0637	0.6296	0.9713	0.9995
	5	6.46	0.1883	0.6438	0.9293	0.9934
	3	8.06	0.5223	0.8103	0.9479	0.9901
	2	8.93	1.0000	1.0000	1.0000	1.0000
0.95	n	t/s	2	4	6	8
	25	0.07	1.0000	1.0000	1.0000	1.0000
	15	0.5	1.0000	1.0000	1.0000	1.0000
	14	2.9	0.6512	0.9991	1.0000	1.0000
	10	4.9	0.0458	0.6069	0.9721	0.9996
	9	5.02	0.0637	0.6296	0.9713	0.9995
	5	6.46	0.1883	0.6438	0.9293	0.9934
	4	8.06	0.2095	0.5682	0.8501	0.9669
	3	8.93	0.4522	0.7419	0.9100	0.9768
0.99	n	t/s	2	4	6	8
	20	0.07	1.0000	1.0000	1.0000	1.0000
	15	0.5	1.0000	1.0000	1.0000	1.0000
	14	2.9	0.6512	0.9991	1.0000	1.0000
	11	4.9	0.0267	0.5405	0.9636	0.9994
	9	5.02	0.0637	0.6296	0.9713	0.9995
	5	6.46	0.1883	0.6438	0.9293	0.9934
	5	8.06	0.0729	0.3623	0.7275	0.9303
	4	8.93	0.1535	0.4606	0.7609	0.9269

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